The Local Fractal Properties of the Financial Time Series on the Polish Stock Exchange Market

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Abstract

We investigate the local fractal properties of the financial time series based on the evolution of the Warsaw Stock Exchange Index (WIG) connected with the largest developing financial market in Europe. Calculating the local Hurst exponent for the WIG time series we find an interesting dependence between the behavior of the local fractal properties of the WIG time series and the crashes appearance on the financial market.

The models and mechanisms enabling to predict the future behavior of the financial market on a long or short term period are a big challenge in the financial engineering and very recently also in econophysics. In the latter case one believes that the approach based on the analogy of the financial market with the complex dynamical system could be very fruitful [1]–[8]. In particular, the scale invariance of the complex systems is used to reveal the log-periodic oscillations characteristic for these systems before the phase transition point is reached. The log-periodic oscillations preceding the crashes or ruptures points, *i.e.* the moments when the increasing long-term trend is being broken and starts to reverse, have really been observed for the market indices or share prices (see *e.g.* [1, 2], [9]–[13]). A number of crash moments t_c has been predicted in the literature so far, some of them being in a very good agreement with the actual moment the crash took place [11]–[13].

However one has to remember that any financial system should be considered as an open system, while the scaleless behavior and the quantitative description following from this assumption are completely true only in a case of the closed statistical systems. Therefore one should be aware that the methods based on the closed system assumption are very sensitive to the number of data points (information) one takes into account to make the fit of log-periodic oscillation parameters. As a result the predictive power of such models can be in general very limited [14].

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Few years ago another approach based on the local properties of the time series was proposed [15]. The method uses the local Hurst exponent H_{loc} [16, 17] or the local fractal dimension D_{loc} of the time series built on the index values or share prices. These quantities are linked together by the well known relation

$$D_{loc} = 2 - H_{loc} \tag{1}$$

in an analogy with the similar equation satisfied for the global D and H values usually used to describe the monofractal signals.

In Ref. [15] it was shown that the local Hurst exponent calculated for the main Dow Jones (DJIA) index forms the characteristic pattern before any of the crashes on American stock market. The H_{loc} values drop significantly down before any rupture point. The moving average $\langle H_{loc}(t)\rangle_5$ of the local Hurst exponent calculated on the one week period (5 sessions) drops down to 0.45 or even less for sessions immediately preceding the rupture point, thus revealing the presence of the growing antipersistence in the financial time series signal before the crash occurs. The similar qualitative behavior of the local H values for other financial time series (shares) has also been confirmed [18].

The advantage in the use of H_{loc} over other methods is that it actually measures 'the local state' of the market and therefore it seems to be more resistable to the long-term inaccuracies or distortions coming *e.g.* from the rapid change of the boundary conditions around the financial system. Therefore the H_{loc} method might be also applied to an open complex system and the financial market is a good candidate of such system.

In this report we intend to apply the technique used in Ref. [15] to investigate the market index of the largest emerging market in Europe - the Warsaw Stock Exchange Index (WIG) incorporating more than 200 companies. It has already 15-years old history up to now with more than 3600 closure day values at the moment.

The WIG time series has been analyzed by us with the Detrended Fluctuation Analysis (DFA) technique [16] to extract the scaling Hurst exponent H. This technique is well described in the literature [16, 17, 19] so we will not quote it here. Let us remind however its local version applied in our calculations.

We first form for a given trading day $t = i$ a time subseries of length N with points in the period $\langle i - N + 1, i\rangle$. We call this subseries the observation box or the observation time-window. Then the standard DFA procedure is applied to this time-window, *i.e.* we cover the subseries with smaller non-overlapping boxes of size τ starting from the given trading point $t = i$ and going backwards in time up to $t = i - N + 1$. In order to cover the whole time-window with τ-size boxes we put the last box in the period $\langle i-N+1, i-[N/\tau]\tau+1\rangle$, where [.] means the integer part. This box partly overlaps the preceding one but this does not modify the obtained results. In each box the detrended signal is found according to DFA method for the simplest linear trend assumed in every box of size τ. The detrended signal fluctuates and its variance $\langle F^2(\tau)\rangle$ is related to the box width τ by the power law relation known for DFA:

$$\langle F^2(\tau)\rangle \sim \tau^{2H_{loc}} \tag{2}$$

Thus moving the observation box of length N session after session we are able to reproduce the whole history of $H_{loc}(t)$ changes in time.

It is well known that the H exponent measures the level of persistence ($H > 1/2$) or anti persistence ($H < 1/2$) in the signal. For $H = 1/2$ one obtains the Brownian (integer) signal with null autocorrelations. Hence, only the case $H \neq 1/2$ is meaningful for practical applications. The observation of the $H_{loc}(t)$ evolution may suggest in what state the financial market is at the given moment. Indeed, big investors usually called speculators cash their profits more frequently if they 'feel' the rupture point in the increasing trend is coming. The more nervous behavior of speculators gives a signal to all other players on the market (spectators) who also start to cash their investments at lower prices. It actually leads after some time to the change of trend and if the market is particularly nervous - to the crash appearance. Thus we put a hypothesis that the nervous situation on the market can (should) be observed as the appearance of the anti correlations in price returns of various assets and finally we may expect the growing local fractal dimension of the financial time series (or decreasing H_{loc}) before a crash.
To check if this hypothesis works well for the polish financial market we studied three main crashes (or rupture points) that have already taken place during the whole WIG history 1992-2007. First we calculated the H_{loc} for any point of the closure day WIG time series. The H_{loc} are usually widely spread out due to the statistical noise. Therefore the moving average $\langle H_{loc}\rangle_5$ of last five sessions (one trading week) or the moving average $\langle H_{loc}\rangle_{21}$ of last 21 sessions (one trading month) have been calculated for the local Hurst exponent for the whole WIG history. The results of $\langle H_{loc}\rangle_{21}$ are shown in Fig. 1 and Fig. 2. All plots were done for the observation boxes of length from $N = 215$ till $N = 300$ sessions. It corresponds to the depth of looking back in time from 10 months till 15 months.
Let us notice that the main trend pattern of $H_{loc}(t)$ is independent on the observation box length N but the depth of H_{loc} fluctuations does depend on N (see Fig. 2). It is because of the different noise cut level changing with the time-window length. To examine the structure of a local behavior more rigorously we have chosen the 10 months observation box to perform further analysis (the similar size was used in Ref. [15]). The examples of plots $F^2(\tau)$ vs τ in log-log scale, from which the H_{loc} values have been extracted according to Eq. (2), are shown in Fig. 3.

Table 1: The main crashes on the Polish financial market. The total percentage drop in WIG and its duration time is shown as well as the percentage drop in the first three sessions after the rupture point.

Date	Initial 3 sessions drop	Total relative drop(duration time)
17.03.94	11 %	65 % (41 sessions)
22.07.98	4 %	39 % (30 sessions)
12.05.06	5 %	21% (24 sessions)

Then the most spectacular crashes on the Polish stock exchange market were examined within this technique in some surrounding of the crash (rupture) points. The investigated cases are found between the vertical lines in Fig. 1. and correspond to crashes described in Table 1. The zoomed evolution of the $H_{loc}(t)$ for these crashes is shown in Fig. 4a-c. The decreasing trend in H_{loc} is the most evident from these plots and lasts for many sessions

before the rupture point occurs. Looking at the common characteristic pattern of H_{loc} plots before the crash moment, we may formulate the following necessary conditions to be *simultaneously* satisfied (*signal to sell*) if the rupture point is expected soon:

1. $H_{loc}(t)$ is in decreasing trend and $\langle H_{loc}\rangle_5 < \langle H_{loc}\rangle_{21}$ except for small fluctuations
2. $\langle H_{loc}\rangle_{21} \leq 0.5$
3. $\langle H_{loc}\rangle_5 \lesssim 0.45$
4. minima of $H_{loc}(t)$ for not necessary consecutive sessions satisfy $H_{loc}^{min}(t) \lesssim 0.4$

Contrary, if all the above conditions are not satisfied we expect the strong *signal to buy* on the market.
Moreover we are able to find a relation between the rate of the $\langle H_{loc}\rangle_5$ drop and the total correction the WIG index gains after the crash. Generally, the deeper the bottom of $H_{loc}(t)$ signal the bigger crash or major correction can be expected. The latter correction can be calculated as the difference in the index signal magnitude between the rupture point and the minimum in the signal value after the crash from which the next long-lasting trend is being formed. We observe this correction is proportional to the absolute slope of the H_{loc} trend assumed as a straight line. Such straight line-fits to the local Hurst exponents have been drawn in Fig. 4a-c for periods immediately preceding the crashes on the Polish stock exchange market. We have also drawn in Fig. 5 a relation between the magnitude of the relative WIG index correction and the slope of the $H_{loc}(t)$ linear trend fit before the crash. Amazingly, this relation is linear! The fit done with just three points may not of course entitle to draw a strong conclusion, nevertheless the relation is striking and worth further investigation.
Finally we checked also the current situation on the market. The plot of the recent WIG signal with the corresponding local fractal properties is illustrated in Fig. 6. The $H_{loc}(t)$ started to fall down almost three trading months ago (from the session ♯3561), despite the WIG signal has still been increasing. The $\langle H_{loc}\rangle_5$ average reached its critical value 1/2 in the end of June (June 28 '07 – session ♯3600), however the crash pattern (conditions (1)-(4)) is not so clear as in previous cases (a)–(c). We still have $\langle H_{loc}\rangle_{21} \sim 1/2$ remaining only slightly below the critical value 1/2. Also $H_{loc}(t)$ minima are higher then previously. As a result a crash has not been formed so far but the 7% correction in WIG signal actually took place. The future situation on the market will, in our opinion, be driven by the forthcoming behavior of $H_{loc}(t)$ as indicated in Fig.7. One may think about two possible scenarios. In the first one the local Hurst exponent will remain in the decreasing trend. If the dropping rate of $H_{loc}(t)$ remains the same as it is now we may expect $\langle H_{loc}(t)\rangle \sim 0.4$ around the beginning of September '07 (see Fig.7). This means that a major crash should take place no later than mid-September. If however, we would observe that $\langle H_{loc}\rangle$ began to increase and the other conditions (1)–(4) were not fulfilled, the crash should not take place and the current drop of WIG index will be just a minor correction in the long-lasting increasing trend of WIG signal. We believe the further $H_{loc}(t)$ evolution will choose very soon one of these scenarios.

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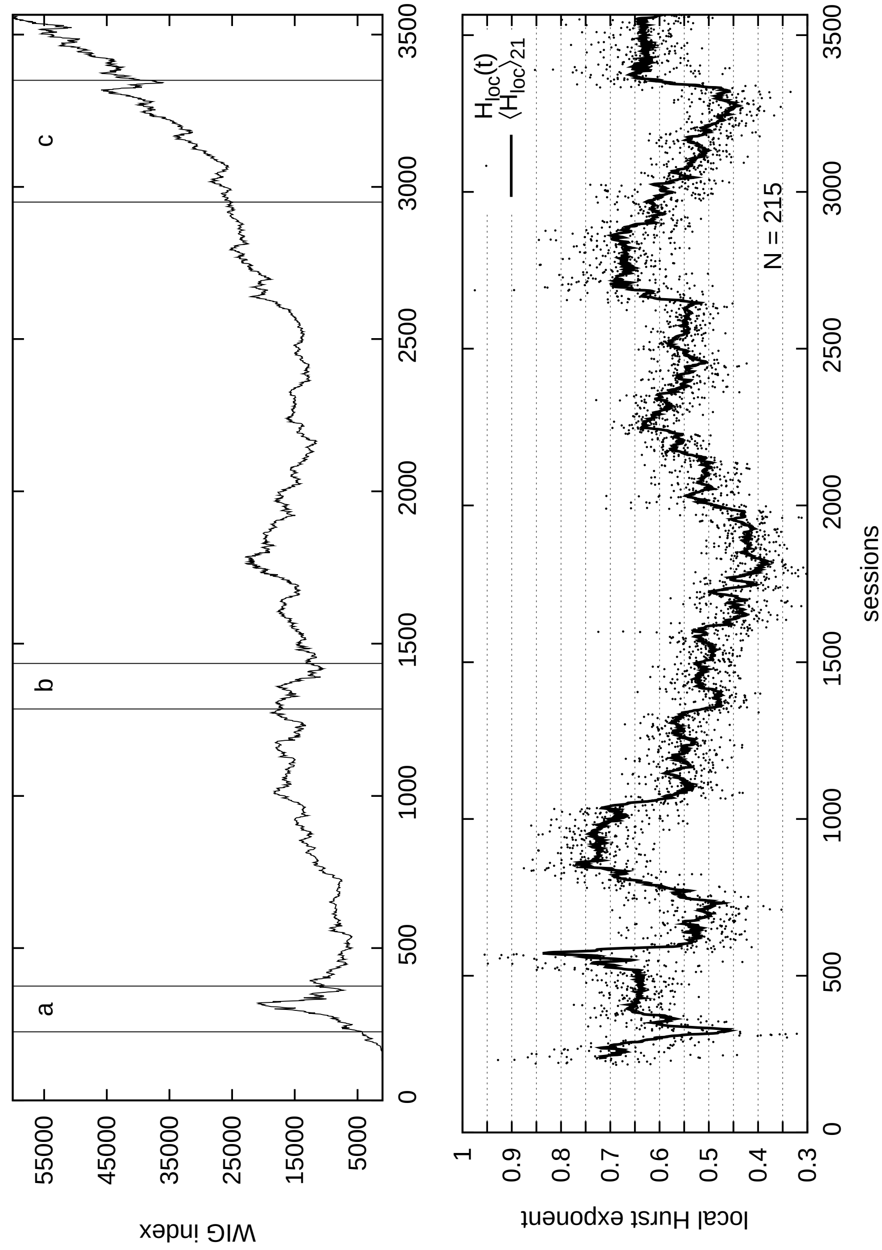

Figure 1: The closure day WIG history April'92-May'07 (top) and the corresponding local Hurst exponent (bottom). The time-dependent Hurst exponent has been calculated in the observation box of $N = 215$ sessions. The solid line represents the moving average of H_{loc} (marked as dots) calculated for one trading month back (21 sessions). Three main crash periods are marked within the vertical lines as (a), (b), (c).

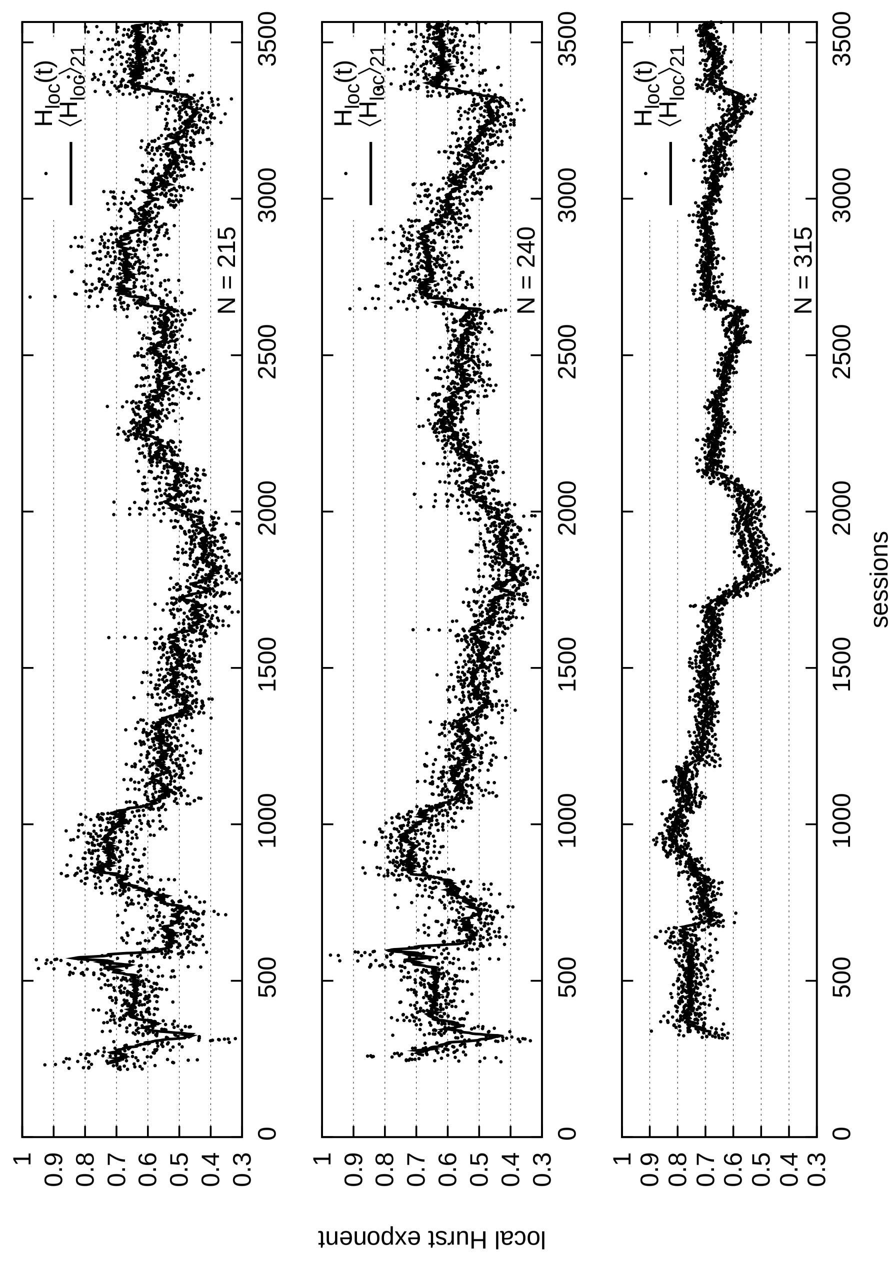

Figure 2: Comparison of the local Hurst exponents obtained for three different observation boxes of length $N = 215, N = 240$ and $N = 315$ sessions respectively. The solid line represents the 1-month moving average of $H_{loc}(t)$

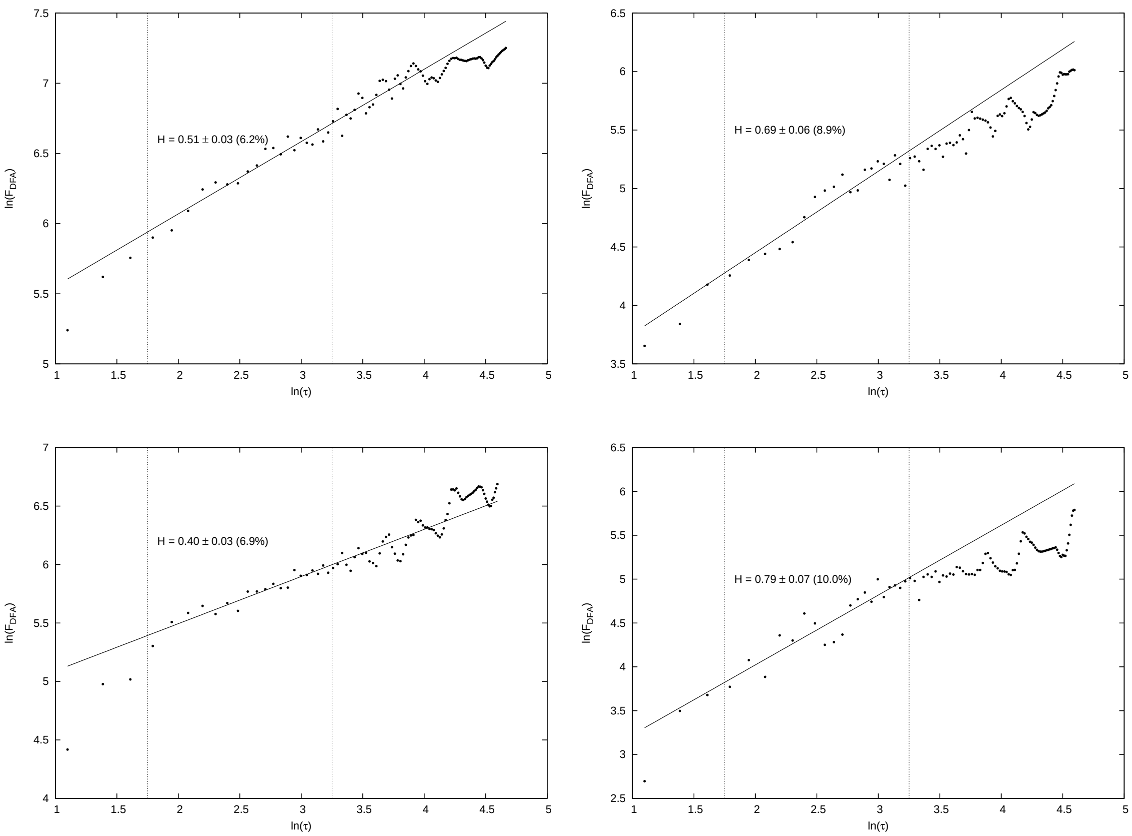

Figure 3: Examples of the local Hurst exponent calculated for the observation box of length $N = 215$ (10 trading months). Plots show the dependence between fluctuation (variance) of the detrended WIG signal in a box of size τ and the size of the box in a log-log scale. The dotted vertical lines mark the scaling range for the power-law from Eq. (2).

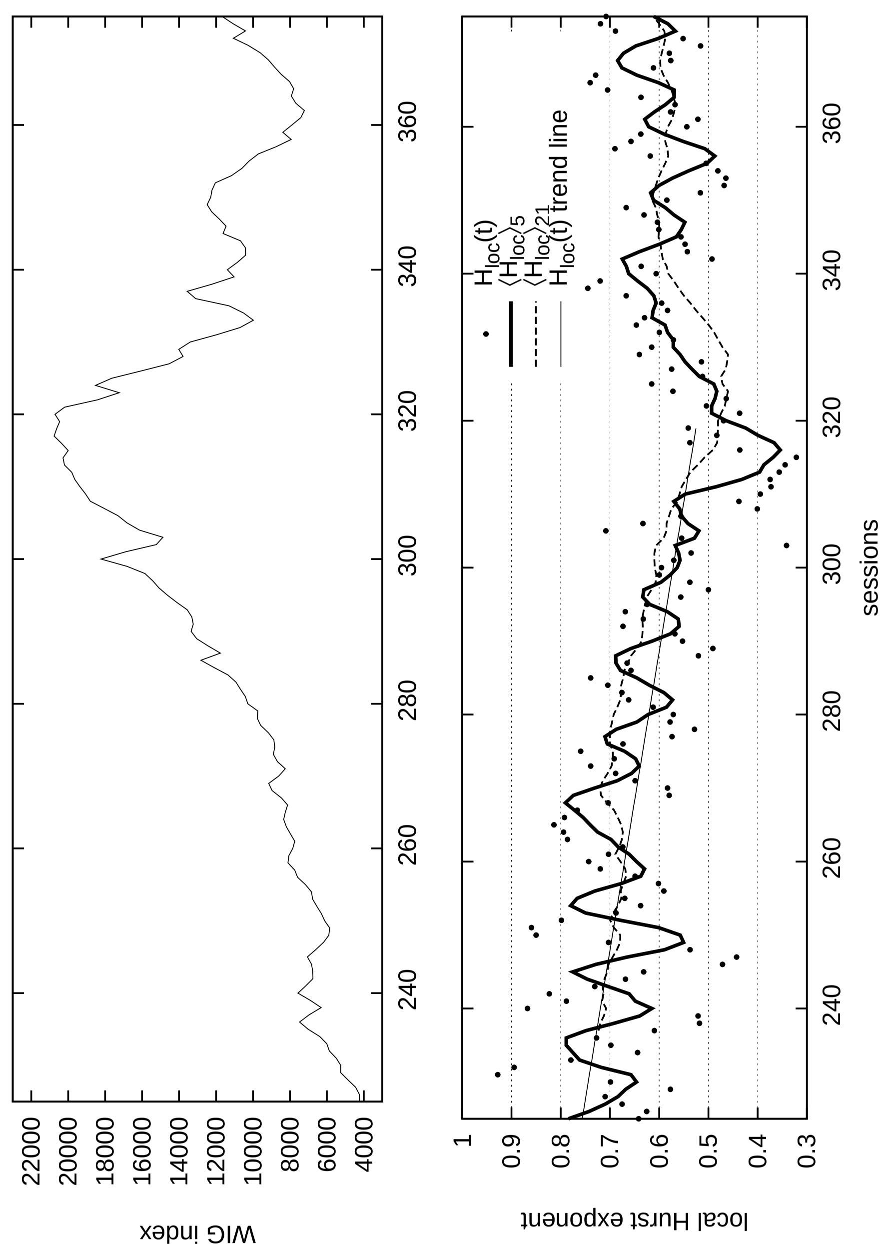

Figure 4a: The 'X-ray' of major crashes (rupture points) on the Polish market done with the local Hurst exponent - March'94. The dots represent H_{loc} values, solid lines indicate the one week $\langle H_{loc}\rangle_5$ and one month $\langle H_{loc}\rangle_{21}$ moving averages of $H_{loc}(t)$. The line-fit of the decreasing trend for $H_{loc}(t)$ before the crash is also marked.

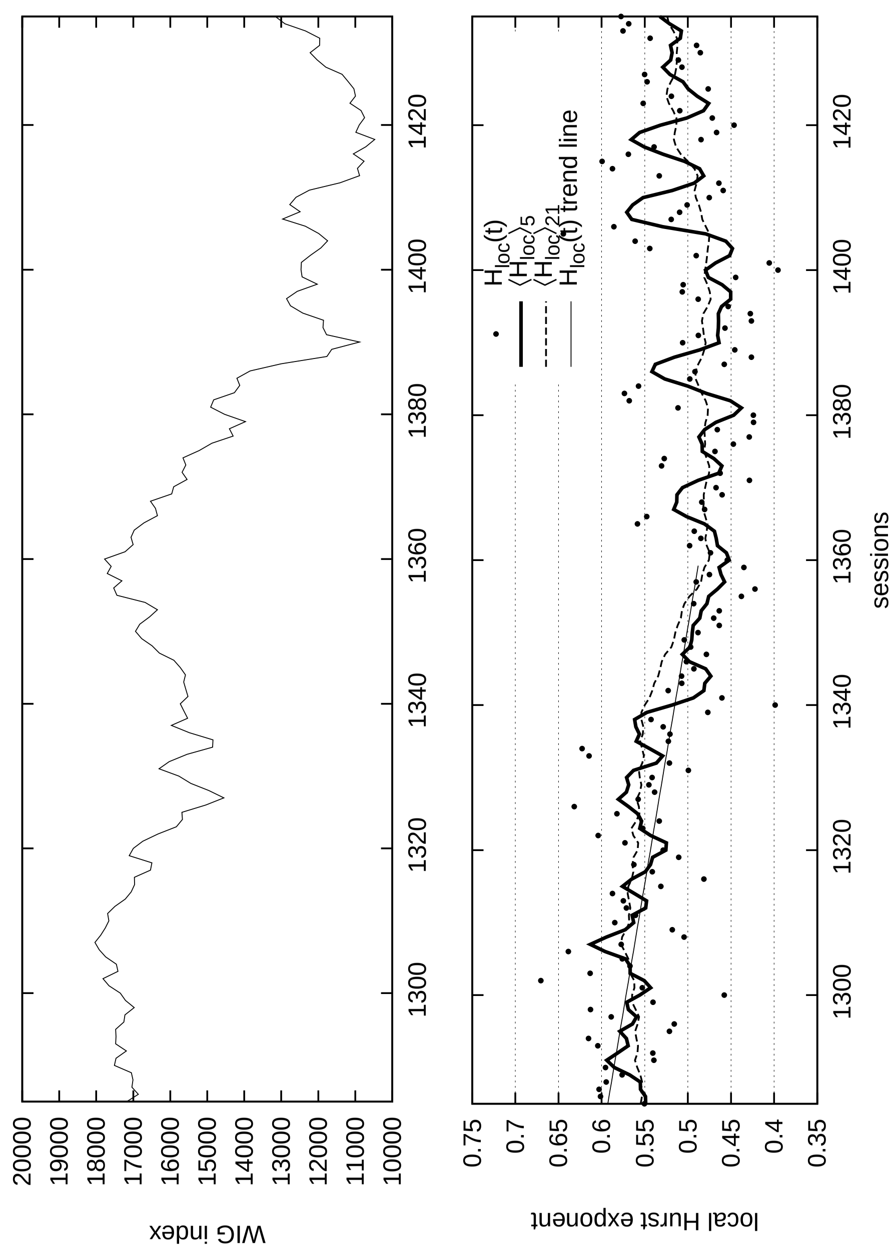

Figure 4b: The 'X-ray' of major crashes (rupture points) on the Polish market done with the local Hurst exponent - July'98. The dots represent H_{loc} values, solid lines indicate the one week $\langle H_{loc} \rangle_5$ and one month $\langle H_{loc} \rangle_{21}$ moving averages of $H_{loc}(t)$. The line-fit of the decreasing trend for $H_{loc}(t)$ before the crash is also marked.

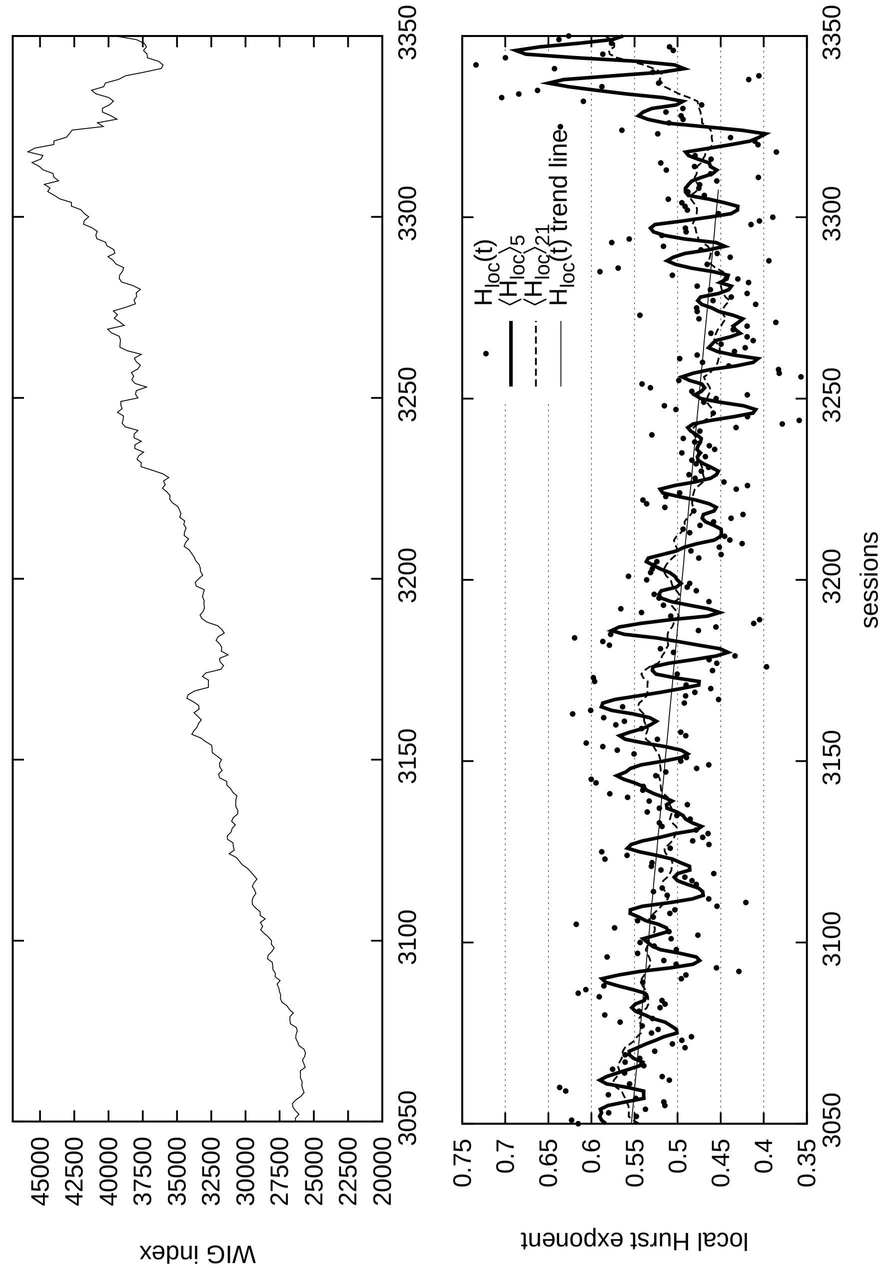

Figure 4c: The 'X-ray' of major crashes (rupture points) on the Polish market done with the local Hurst exponent - May'06. The dots represent H_{loc} values, solid lines indicate the one week $\langle H_{loc}\rangle_5$ and one month $\langle H_{loc}\rangle_{21}$ moving averages of $H_{loc}(t)$. The line-fit of the decreasing trend for $H_{loc}(t)$ before the crash is also marked.

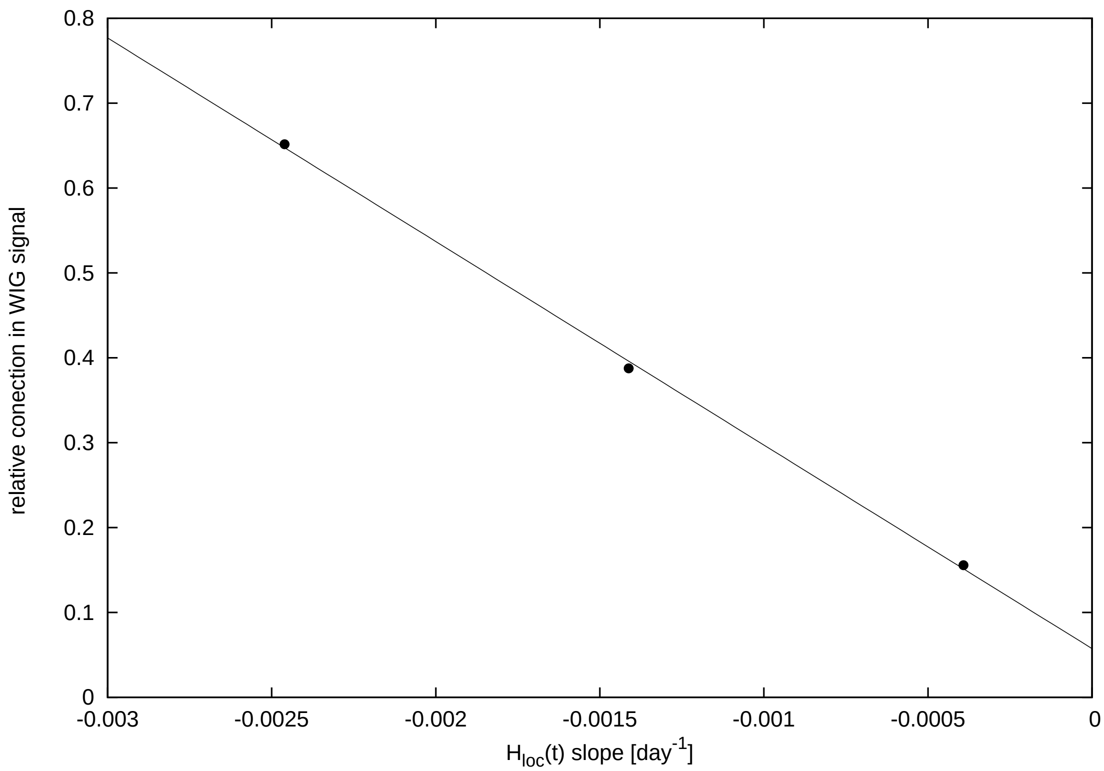

Figure 5: The dependence between the magnitude of correction in WIG signal after the crash and the slope of line-fit to the $H_{loc}(t)$ trend.

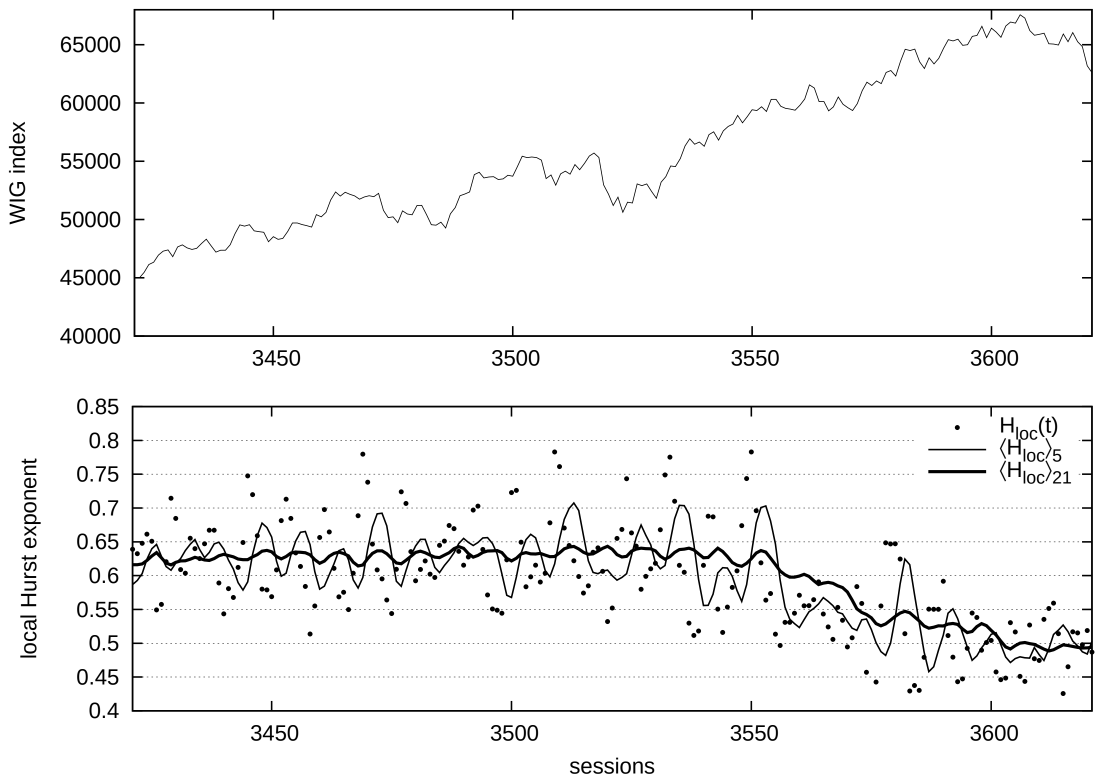

Figure 6: The recent situation on the Polish market and the $H_{loc}(t)$ evolution.

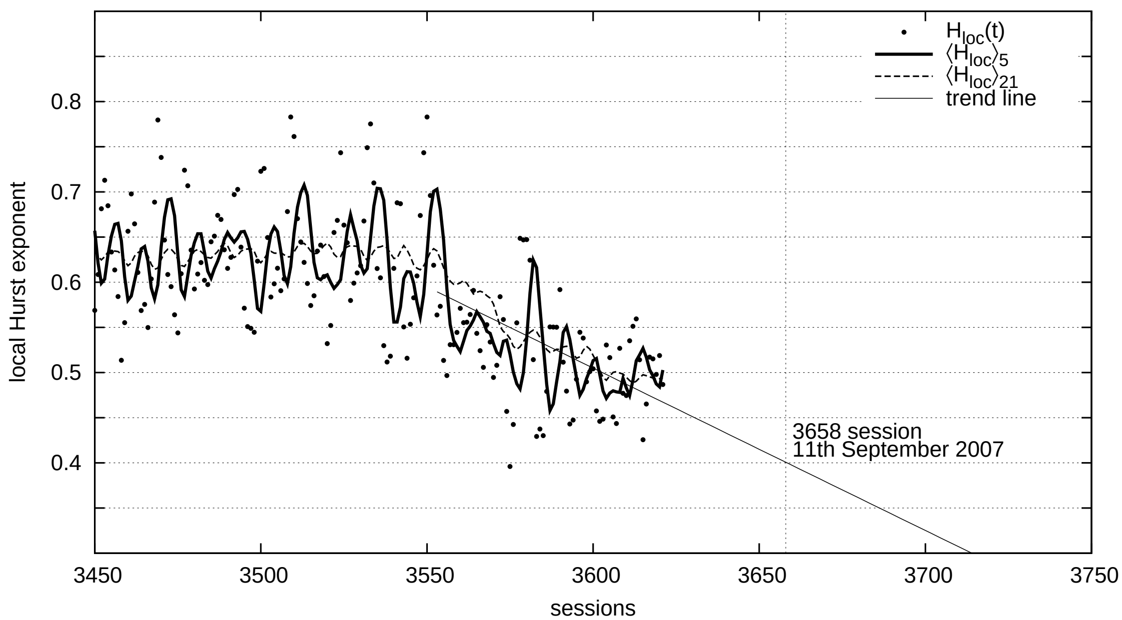

Figure 7: The possible scenario of the further $H_{loc}(t)$ evolution if the decreasing trend in local Hurst exponent will be kept. The data taken into account terminate on July 27 '07. The straight line fit to $H_{loc}(t)$ was done for 3 trading months back.